Topological Synthesis of Non–Reciprcoal Networks

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1. INTRODUCTION

Combinatorial topology is useful for handling various problems not only in analysis [1)~21) 42)~45) 48) 49)] but also in synthesis. [22)~24) 47)] This paper deals with topological synthesis of non-reciprocal network specified by the so-called driving admittance matrix. Objects which determine the network are the connection⁺ of it and element inpedances thereof, and then it follows that we can algebraically determine the node-branch incidence matrix ⁺⁺ and the branch impedance matrix, so that transfer admittance matrix of the network is equal to the given matrix. This paper chiefly deals with the topological method of determing the node-branch incidence matrix.

When we denote the branch e. m. f. matrix by e_λ and the branch current matrix by i^κ,

$$i^\kappa = C^{\kappa\lambda}e_\lambda\,, \qquad (1)$$

where κ or λ is the notation for an arbitrary branch, namely the number $1,\cdots\cdots,n$, and is called an index⫩. This $C^{\kappa\lambda}$ is called "the transfer admittance matrix".

When we denote the node-branch incidence matrix by D^h_κ, where h is an index for a point, by the first Kirchhoff's law we have a relation

$$D^h_\kappa i^\kappa = O. \qquad (2)$$

Therefore by (1) and (2)

$$D^h_\kappa C^{\kappa\lambda}e_\lambda = O.$$

⁺ Concerning the foundations of network topology, see Ref. 25) ~ 34) etc.

⁺⁺ "The node-branch incidence matrix"[2)] also is called "1-incidence matrix"[25)], or "incidence matrix."[11)] in short.

⫩ Notations in this paper are based on tensor geometry [39) 40) 41)], but the reader may regard them as a mere matrix expression. The letter with two indices is a matrix and the letter with one index is a one row or one column matrix. Namely, in this index notation, this expression substitutes the arbitrary elements for the matrix. As for the product of matrices, we adopt the summation convention called the Einstein's summation convention: if an index appears twice in the same term once as a subscript and once as a superscript, the sign $\sum$ will be omitted.

In order to be satisfied for any e_λ, it must hold that

$$D^h_\kappa C^{\kappa\lambda} = O. \qquad (3)$$

Then D^h_κ is determined by a linear combination of a maximal set of independent solutions x_κ for

$$x_\kappa C^{\kappa\lambda} = O. \qquad (4)$$

If we denote a maximal set of them by x^b_κ ++,

$$D^h_\kappa = K^h_b x^b_\kappa, \qquad (5)$$

where b is an index for the sets of independent solutions.

And from the characteristics of $C^{\kappa\lambda}$, it also holds that

$$C^{\kappa\lambda} x_\lambda = O. \qquad (6)$$

(In regard to Equ. (6), see the following description.)

In case that the driving admittance $C^{\kappa\lambda}$ is given, therefore, by (4) and (5) we can determine the connection specified by the values of this $C^{\kappa\lambda}$. If D^h_κ determined by (4) and (5) satisfies the character of a node-branch incidence matrix, by this D^h_κ we can determine the connection directly.

If not, by the part of D^h_κ which satisfies the character we determine the connection and by the other part we determine the use of ideal transformers.

Therefore whether ideal transformers are necessary or not, is determined by the fact whether D^h_κ given by a linear combination of x^b_κ satisfies the character of a node-branch incidence matrix or not.

After the connection and the ideal transformers are determined, we can determine branch impedances by comparing the given matrix with the driving admittance matrix calculated from the newly determined connection and the unknown branch admittances.

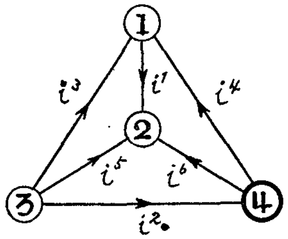
2. THE GENERAL CONSIDERATION OF A. C. NETWORKS

In this chapter, the author will describe the general outline of A. C. networks and introduce a few algebraic treatments of them preliminary to a topological consideration of network synthesis.

2. 1 Node-branch incidence matrix

First, we denote the node-branch incidence matrix by D^h_κ. D^h_κ is $\alpha^0 \times \alpha^1$ – matrix, where α^0 is the number of nodes and α^1 the number of branches. For example, in Fig. 1 the node-incidence matrix is[2)]

++ x^b_κ is a $m \times n$ matrix, where $m = n - k$, $n = \dim. (T^{\kappa\lambda})$ and $k = \text{rank} (T^{\kappa\lambda})$.



$$D^h_\kappa = \begin{array}{|cccccc|} D^1_1 & D^1_2 & D^1_3 & D^1_4 & D^1_5 & D^1_6 \\ D^2_1 & D^2_2 & D^2_3 & D^2_4 & D^2_5 & D^2_9 \\ D^3_1 & D^3_2 & D^3_3 & D^3_4 & D^3_5 & D^3_6 \\ D^4_1 & D^4_2 & D^4_3 & D^4_4 & D^4_5 & D^4_9 \end{array} = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & \cdot & -1 & -1 & \cdot & \cdot \\ 2 & -1 & \cdot & \cdot & \cdot & -1 & -1 \\ 5 & \cdot & 1 & 1 & \cdot & 1 & \cdot \\ 4 & \cdot & -1 & \cdot & 1 & \cdot & 1 \end{array} \quad (7)$$

Fig. 1

Obviously the column corresponding to each branch has both of exactly one "+1" and exactly one "−1" as elements, and all the other elements are zero. Then one row of D^h_κ linearly depends on the others of D^h_κ. Therefore if we denote D^h_κ without an arbitraiy row by D^h_κ, it is frequently sufficient that we use D^a_κ instead of D^h_κ. and we also call D^a_κ "the node-branch incidence matrix" .Here the above indices are used in the following sections as follows:

all nodes:	h, i	$= 1, 2, \cdots\cdots, m_1+1,\ m_1+1 = \alpha^o$,
independent nodes:	a_1, b_1	$= 1, 2, \cdots\cdots, m_1$,
branches:	κ, λ	$= 1, 2, \cdots\cdots, n,\ n = \alpha^1$.

For example, in Fig. 1 D^a_κ is given as follows:

$$D^a_\kappa = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 1 & \cdot & -1 & -1 & \cdot & \cdot \\ 2 & -1 & \cdot & \cdot & \cdot & -1 & -1 \\ 3 & \cdot & 1 & 1 & \cdot & 1 & \cdot \end{array} \quad . \qquad (8)$$

Aglebraically, the following is always the condition which is necessary and sufficient for determination of D^a_κ: ⌈the column corresponding to each branch has either a single "+1" or a single "−1" or it may have both, and all the other elements are zero⌋.

This character is very important to our topological synthesis.

If current sources do not exist, the first Kirchhoff's law is denoted by

$$D^h_\kappa i^\kappa = 0 \qquad \text{or} \qquad D^a_\kappa i^\kappa = 0. \qquad (9)$$

For example, in Fig. 1, it is

$$\begin{bmatrix} 1 & \cdot & -1 & -1 & \cdot & \cdot \\ -1 & \cdot & \cdot & \cdot & -1 & -1 \\ \cdot & 1 & 1 & \cdot & 1 & \cdot \end{bmatrix} \begin{bmatrix} i^1 \\ i^2 \\ i^3 \\ i^4 \\ i^5 \\ i^6 \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}.$$

2.2 **Ideal transformers** [35)]

With respect to ideal transformers which have no magnetic leakage flux and have the pemeance of infinite magnitude, there holds the following relation:

$$N_{\kappa}^{a2} i^{\kappa} = 0, \tag{10}$$

where each element of N_{κ}^{a2} is the number of turns of a winding coil κ in each magnetic core a_2 of ideal transformers, and the sign of each number is either positive or negative according to the right or the left sense of the oriented branch for the oriented core. N_{κ}^{a2} is a $t \times \alpha^1$ -matrix, where t is the number of ideal transformers, and it is called "the winding matrix" or "the turn-ratio matrix". Namely Equ. (10) shows the relation that the sum of the magneto-motive forces in the core is zero.

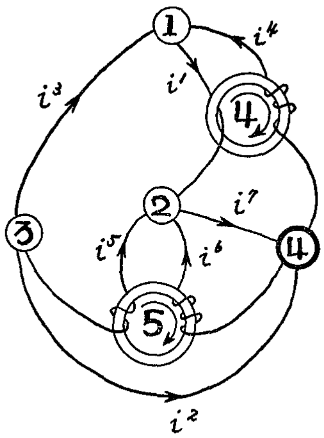


Fig. 2

Here $a_2, b_2, = m_1 + 1, \cdots\cdots, m_1 + t$
$= m_1 + 1, \cdots\cdots, m,$
where t is the number of ideal transformers. For example, in Fig. 2

$$N_{\kappa}^{a2} = \begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 4 & -1 & \bullet & \bullet & 4 & \bullet & \bullet & \bullet \\ 5 & \bullet & \bullet & \bullet & \bullet & 2 & 3 & \bullet \\ \hline \end{array} \,. \tag{11}$$

[Each element of a winding matrix is a real integer being positive or negative.]
Now, defining that

$$B_{\kappa}^{a} = \begin{bmatrix} D_{\kappa}^{a1} \\ \hline N_{\kappa}^{a2} \end{bmatrix} = \begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 1 & \bullet & -1 & -1 & \bullet & \bullet & \bullet \\ 2 & -1 & \bullet & \bullet & \bullet & -1 & -1 & 1 \\ 3 & \bullet & 1 & 1 & \bullet & 1 & \bullet & \bullet \\ \hline 4 & -1 & \bullet & \bullet & 4 & \bullet & \bullet & \bullet \\ 5 & \bullet & \bullet & \bullet & \bullet & 2 & 3 & \bullet \\ \hline \end{array} , \tag{12}$$

B_{κ}^{a} is a $(\alpha^o - 1 + t) \times \alpha^1$-matrix, and from Equ. (9) and (10) we have

$$B_{\kappa}^{a} i^{\kappa} = 0. \tag{13}$$

Here rank of D_{κ}^{a1}, N_{κ}^{a2} and B_{κ}^{a} respectively are given by

$$\rho_D = \rho(D_{\kappa}^{h}) = \rho(D_{\kappa}^{a1}) = \alpha^o - 1,$$
$$\rho_N = \rho(N_{\kappa}^{a2}) = t,$$
and $$\rho_B = \rho(B_{\kappa}^{a}) = \rho_D + \rho_N = \alpha^o - 1 + t,$$

where it is postulated that D_{κ}^{a1} and N_{κ}^{a2} mutually are linearly-independent: because any winding matrix N_{κ}^{a2} that depends linearly on the node-branch incidence matrix D_{κ}^{a1} is not worthy of use. B_{κ}^{a} is called "the generalized node-branch incidence matrix".

2.3 The generalized loop matrices[13)]

Let C_p^{κ} be a $(\alpha^1 - \rho_B) \times \alpha^1$-matrix formed from a maximal set of linear-independent solutions x^κ for the equation

$$B^a_\kappa x^\kappa = 0 . \tag{14}$$

Provided that C^κ_p is the maximal set of the independent solutions, where $p,q = m''+1'', \cdots\cdots, m''+k''$,

$$B^a_\kappa C^\kappa_p = 0 \quad \text{or} \quad C^\lambda_q B^b_\lambda . \tag{15}$$

C^κ_q is a $(\alpha^1 - \alpha^0 + 1 - t) \times \alpha^1$ -matrix and, for example, in Fig. 2 the relation (14) is

$$B^a_\kappa x^\kappa = \begin{bmatrix} 1 & \cdot & -1 & -1 & \cdot & \cdot & \cdot \\ -1 & \cdot & \cdot & \cdot & -1 & -1 & 1 \\ \cdot & 1 & 1 & \cdot & 1 & \cdot & \cdot \\ \hline -1 & \cdot & \cdot & 4 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 2 & 3 & \cdot \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \\ x^7 \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \end{bmatrix} .$$

The one example of C^κ_p is

$$C^\kappa_p = \begin{array}{c|cc|} & 6 & 7 \\ \hline 1 & 4 & \cdot \\ 2 & -3 & 12 \\ 3 & 3 & \cdot \\ 4 & 1 & \cdot \\ 5 & \cdot & -6 \\ 6 & \cdot & 8 \\ 7 & 4 & -4 \end{array} .$$

B^a_κ and C^p_κ are zero-factors of each other, and if ideal transformers do not exist, B^a_κ equals D^a_κ (D^{a1}_κ) and C^κ_p can be equal to a loop-branch incidence matrix R^κ_p. ≠≠≠

For example, in Fig. 1 as we put

$$B^a_\kappa = D^a_\kappa = D^{a1}_\kappa = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & \cdot & -1 & -1 & \cdot & \cdot \\ 2 & -1 & \cdot & \cdot & \cdot & -1 & -1 \\ 3 & \cdot & 1 & 1 & \cdot & 1 & \cdot \end{array} ,$$

the maximal set C^κ_p of indepenent solutions x^κ for the equation

$$B^a_\kappa x^\kappa = D^a_\kappa x^\kappa = D^{a1}_\kappa x^\kappa$$

$$= \begin{bmatrix} 1 & \cdot & -1 & -1 & \cdot & \cdot \\ -1 & \cdot & \cdot & \cdot & -1 & -1 \\ \cdot & 1 & 1 & \cdot & 1 & \cdot \end{bmatrix} \begin{bmatrix} x^1 \\ x^2 \\ x^3 \\ x^4 \\ x^5 \\ x^6 \end{bmatrix} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

is determined as follows:

≠≠≠ D^{a1}_κ and R^κ_p respectively correspond to div. and rot. in a vector field, and it is because of this that we use specifically the letter D and R to denote the corresponding matrices.

$$C_\kappa^p = \begin{array}{c} \begin{array}{ccc} 4 & 5 & 6 \end{array} \\ \left|\begin{array}{ccc} 1 & -1 & \bullet \\ \bullet & \bullet & -1 \\ 1 & \bullet & \bullet \\ \bullet & -1 & \bullet \\ -1 & \bullet & 1 \\ \bullet & 1 & -1 \end{array}\right| \end{array} = R_\kappa^p$$

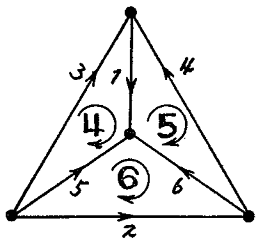


Fig. 3

Evidently this matrix is a loop-branch incidence matrix [2] when independent loops are chosen such as in Fig. 3. And the aforementioned relation is denoted by

$$D_\kappa^\alpha R_\kappa^p = 0 \quad \text{or} \quad R_p^\lambda D_\lambda^b = 0. \tag{16}$$

Namely D_κ^α and R_p^κ are zero-factors of each other.

By Equ. (13) and (15), i^κ can be a linear combination of C_p^κ. Then if J^p is the adequate matrix of one column which is a set of coefficients of the above combination, we have a relation

$$i^\kappa = C_p^\kappa J^p. \tag{17}$$

This J^p is called "the generalized loop current matrix", and if $B_\kappa^\alpha = D_\kappa^\alpha$ i. e. $C_p^\kappa = R_\kappa^p$, then J^p is an ordinal loop current matrix.

Rank of C_p^κ is equal to

$$\rho_c = \alpha^1 - \rho_B = \alpha^1 - (\alpha^0 - 1 + t)$$
$$= p^1 - t,$$

where α^1 is the number of branches, and p^1 is the first Betti number, which is the number of independent loops.

2.4 The principle of the energy conservation of the generalized second Kirchhoff's law

The loss of power in ideal transformers is zero, and if there is no current source, we may consider only

$\bar{i}^\lambda u_\lambda$ in passive elements,

and $\bar{i}^\lambda e_\lambda$ in voltage sources,

where u_λ is the element voltage drop matrix of one column, e_λ the branch e. m. f. matrix of one column and $\bar{i}^\lambda$ the conjugate matrix of the branch current matrix of one row.

From the general postulate of the conservation of energy, therefore, we get

$$\bar{i}^\lambda u_\lambda - \bar{i}^\lambda e_\lambda = 0,$$
$$\bar{i}^\lambda (u_\lambda - e_\lambda) = 0.$$

By Equ. (17) we have the equation

$$\bar{J}^q \bar{C}_q^\lambda (u_\lambda - e_\lambda) = 0.$$

In oder to make this equation hold for any J^p, we need

$$\bar{C}_q^\lambda (u_\lambda - e_\lambda) = 0, \tag{18}$$
$$\therefore \quad C_q^\lambda (u_\lambda - e_\lambda) = 0, \tag{19}$$
$$\text{i. e.} \quad C_q^\lambda u_\lambda = C_q^\lambda e_\lambda, \tag{20}$$

which is no other than the second Kirchhoff's law if ideal transformers do not exist, because $C_q^\lambda = R_q^\lambda$ in this case. We shall call it "the generalized Kirchhoff's law" and this method of deriving the second Kirchhoff's law from the energy principle "Plank's logic".

2.5 Fundmental equations of a network problem and the transfer admittance matrix[13) 35) 36) 37)]

The fundamental equations of an electrical network can be arranged as follows:

the generalized 1-Kh. law: $B_\kappa^q i^\kappa = 0,$ (21)

the generalized 2-Kh. law: $C_q^\lambda u_\lambda = C_q^\lambda e_\lambda,$ (22)

Ohm's law: $u_\lambda = z_{\lambda\kappa} i^\kappa,$ (23)

where $z_{\lambda\kappa}$ is the branch impedance matrix.

If e_λ is given, u_λ and i^κ are determined by the equation (21), (22) and (23).

When we wish to determine the branch current i^κ caused by branch e. m. f. e_λ, the coefficient $C^{\kappa\lambda}$ of the solution such as

$$i^\kappa = C^{\kappa\lambda} e_\lambda \tag{24}$$

is generally called "the driving admittance matrix".

By means of matrix calculations we may determine it as follows:

by (22) and (23)

$$C_q^\lambda z_{\lambda\kappa} i^\kappa = C_q^\lambda e_\lambda$$

holds, and then as the equation

$$(17): \quad i^\kappa = C_p^\kappa J^p$$

holds, changing the variable i^κ to J^p, we have

$$C_q^\lambda z_{\lambda\kappa} C_p^\kappa J^p = C_q^\lambda e_\lambda,$$

$$J^p = (C_q^\lambda z_{\lambda\kappa} C_p^\kappa)^{-1} C_q^\lambda e_\lambda,$$

and therefore by (17)

$$i^\kappa = C_p^\kappa (C_q^\lambda z_{\lambda\kappa} C_p^\kappa)^{-1} C_q^\lambda e_\lambda. \tag{25}$$

Namely, the driving admittance mtrix is

$$C^{\kappa\lambda} = C_p^\kappa (C_q^\lambda z_{\lambda\kappa} C_p^\kappa)^{-1} C_q^\lambda. \tag{26}$$

If there is no ideal transformer, we may change C_p^κ to R_p^κ, and in this case

$$R^{\kappa\lambda} = R_p^\kappa (R_q^\lambda z_{\lambda\kappa} R_p^\kappa)^{-1} R_q^\lambda. \tag{27}$$

3. TOPOLOGICAL SYNTHESIS OF THE DRIVING ADMITTANCE MATRIX

3.1 The topological method

In the preceding section, we have dermined the distribution of currents

i^κ caused by the given voltage source e_λ about the given network. But frequently we meet with a question to determine a network which has the given distribution of currents i^κ caused by the given voltage source e_λ. Here this problem is to determine the network which specified the given driving admittance matrix $T^{\kappa\lambda}$. Now if the frequency of the voltage sources is constant, we may treat this problem by the topological synthesis of the network specified by the driving admittance matrix given as arbitrary complex numbers, where it is postulated that we do not need the relation

$$C^{\kappa\lambda} = C^{\lambda\kappa}, \tag{28}$$

because we consider the non-reciprocal networks.

It is postulated that passive elements may be dissipative or not.

By Equ. (15) and (26), whatever voltage sources e_λ are, there must hold

$$B^a_\kappa C^{\kappa\lambda} = 0 \quad \text{and} \quad C^{\kappa\lambda} B^b_\lambda = 0, \qquad (29') \ (29'')$$

where the rank of $C^{\kappa\lambda}$ is, from (26),

$$\rho = \rho(C^{\kappa\lambda}) = \rho_c = p^1 - t.$$

The maximal number of independent solutions

$$B^a_\kappa x^\kappa = 0 \tag{30}$$

is

$$\begin{aligned} m &= \alpha^1 - \rho_c \\ &= \alpha^1 - (p^1 - t) \\ &= \alpha^1 - (\alpha^1 - \alpha_0 + 1 - t) \\ &= \alpha^o - p^1 + t, \end{aligned}$$

where x^κ is an unknown nmatrix of one colum.

Now if we denote a maximal set of independent solutions by $x^b_\kappa (b = 1, \cdots, m)$, there holds

$$C^{\kappa\lambda} x^b_\lambda = 0. \tag{31}$$

From (29) and (31), it follows that x^b_λ and B^b_λ are linear combinations of each other.

Hence we may put

$$B^a_\kappa = K^a_b x^b_\kappa , \tag{32}$$

where K^a_b is the adequate transformation matrix, and $\det (K^a_b) \neq 0$.

Therefore we can determine B^a_κ from x^b_κ by a suitable linear transformation, so that B^a_κ may satisfy the character of a node-branch incidence matrix and a winding matrix. In other words, if the rows of B^a_κ do not satisfy the character of a node-branch incidence matrix, we must use ideal transformers. After we have determined the connection and ideal transformers, we determine the values of branch impedances. Namely, we determine the general inverse C^q_κ of C^κ_p satisfying

$$C^q_\kappa C^\kappa_p = A^q_p, \tag{33}$$

where A^q_p is an unit matrix of dimension $k = (\alpha^1 - \rho_B)$, and as C^κ_p generally is a rectangular matrix, namely, a $\alpha^1 \times (\alpha^1 - \rho_B)$ —matrix, C^q_κ is determined not uniquely, but it may be an arbitrary matrix satisfying Equ. (33).

Hence it follows that *

$$
\begin{aligned}
&(C^{q}_{\kappa}C^{\kappa\lambda}C^{p}_{\lambda})^{-1} \\
&= [C^{q}_{\kappa}\{C^{\kappa}_{p}(C^{\lambda}_{q}z_{\lambda\kappa}C^{\kappa}_{p})^{-1}C^{\lambda}_{q}\}C^{p}_{\lambda}]^{-1} \\
&= C^{\lambda}_{q}z_{\lambda\kappa}C^{\kappa}_{p} = z_{qp}. \qquad (34)
\end{aligned}
$$

Then we can determine branch impedance matrix $z_{\lambda\kappa}$ by comparing $(C^{q}_{\kappa}T^{\kappa\lambda}C^{p}_{\lambda})^{-1}$ with $C^{\lambda}_{p}z_{\lambda\kappa}C^{\kappa}_{p}$ as $R^{\kappa\lambda}$, C^{κ}_{p} and C^{q}_{κ} being already known.

Here if we compare them with each other after we put

$$z_{qp} = C^{\lambda}_{q}z_{\lambda\kappa}C^{\kappa}_{p} = (C^{p}_{\kappa}C^{\kappa\lambda}C^{q}_{\lambda})^{-1},$$

$$z_{(qp)} = \frac{1}{2}(z_{qp} + z_{pq}),$$

and $$z_{[qp]} = \frac{1}{2}(z_{qp} - z_{pq}),$$

comparing is done more easily. $z_{(qp)}$ is the symmetrical component of z_{qp}, and $z_{[qp]}$ is skew-symmetrical component of z_{qp},

i. e. $$z_{(qp)} = +z_{(pq)},$$

and $$z_{[qp]} = -z_{[pq]}.$$

$z_{(qp)}$ is realized by use of the ordinary elements auch as resistances, inductances and capacitances, and $z_{[qp]}$ by use of non-reciprocal elements such as gyrators.

3.2 The necessary and sufficient condition for $C^{\kappa\lambda}$.

In this topological method, we can have the necessary and sufficient condition for $R^{\kappa\lambda}$ unless otherwise provided. It is that

"$C^{\kappa\lambda}$ has a real zero-factor on the left and the right sides."

Namely, there must exist the solutions x^{b}_{λ} for $C^{\kappa\lambda}x^{b}_{\lambda} = 0$ and $x^{a}_{\kappa}C^{\kappa\lambda} = 0$, All the elements of this solution must be real numbers. In other words, each dependent row must be a linear combination of other independent rows with coefficients of real numbers, and each dependent column must be the linear combination of other independent columns with the same coefficients of the above.

Only if the given $C^{\kappa\lambda}$ satisfies the above condition**, can we make a

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* C^{p}_{λ} is the transposing matrix of C^{q}_{κ} as matrix expression.

** As in this paper the resistances may be positive or not, it does not directly concern the solution of the important unsolved problem of the necessary and sufficient conditions for realizability by an n-port network of resistances without ideal transformers. And this method is very useful for the consideration of the above problem, but it is so difficult that we will not take up the subject here.

network specified by the given driving admittance matrix $C^{\kappa\lambda}$. This network is shown as both B^{α}_{κ} and $z_{\lambda\kappa}$ algebraically. In order to use the smaller number of ideal transformers, it would be better that the given matrix $C^{\kappa\lambda}$ satisfies the following condition in stead of the previcously mentioned condition. But this also is not a sufficient condition for synthesis without ideal transformers, and further we need the condition that real zero-factor x^{λ}_{k} of $C^{\kappa\lambda}$ can be transformed by a linear transformation so that it may satisfy the character of a node-branch incidence matrix.

Namely, "Elementary minor exists". If the rank of the matrix is k and all the diagonal minor determinants of dim. k and of rank k are invariant, these are called "elemental minors".

In other words, it means that dependent rows or dependent columns are linear combinations of independent ones with coefficients 0, $+1$ and -1.

4. EXAMPLES

4.1 Problem 1

Design a network specified by the driving admittance matrix such as

$$C^{\kappa\lambda} = \frac{1}{162+j38}\ \begin{array}{c} \\ 1\\ 2\\ 3\\ 4\\ 5\\ 6 \end{array}\!\!\begin{array}{c} \begin{array}{cccccc} \mathit{1} & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left|\begin{array}{cccccc} 30+j7 & 16 & 26-j7 & -14-j7 & -4-j14 & -10+j7 \\ 12-j4 & 38+i2 & 40-j8 & 26+j6 & 28-j4 & -2+j10 \\ 4-j7 & 12-j4 & 38-j7 & 8+j3 & 34 & -26+j3 \\ -18-j11 & 22+j2 & 14-j1 & 40+j13 & 32+j10 & 8+j3 \\ -26-j14 & -4-j4 & 12 & 22+j10 & 38+j14 & -16-j4 \\ 8+j3 & 26+j6 & 2-j1 & 18+j3 & -6-j4 & 24+j7 \end{array}\right| \end{array}.$$

SOLUTION A maximal set of independent rows is the first, second and third rows. Dependent relations are similar in the rows and also in the columns, and then this matrix has the same real-zero-factor of rank 3 on the right and the left sides.

$$C^{\kappa\lambda}z_{\lambda} = \frac{1}{162+j38}\left|\begin{array}{cccccc} 30+j7 & 16 & 26-j7 & -14-j7 & -4-j14 & -10+j7 \\ 12-j4 & 38+j2 & 40-j8 & 26+j6 & 28-j4 & -2+j10 \\ 4-j7 & 12-j4 & 38-j7 & 8+j3 & 34 & -26+j3 \end{array}\right| \left|\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array}\right| = 0,$$

then

$$\left|\begin{array}{ccc} 30+j7 & 16 & 26-j7 \\ 12-j4 & 38+j2 & 40-j8 \\ 4-j7 & 12-j4 & 38-j7 \end{array}\right| \left|\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\right| = -\left|\begin{array}{ccc} -14-j7 & -4-j14 & -10+j7 \\ 26+j6 & 28-j4 & -2+j10 \\ 8+j3 & 34 & -26+j3 \end{array}\right| \left|\begin{array}{c} x_4 \\ x_5 \\ x_6 \end{array}\right|,$$

and putting

$$\left|\begin{array}{c} x_4 \\ x_5 \\ x_6 \end{array}\right| = \left\{ \left|\begin{array}{c} 1 \\ \cdot \\ \cdot \end{array}\right|,\ \left|\begin{array}{c} \cdot \\ 1 \\ \cdot \end{array}\right| \text{ or } \left|\begin{array}{c} \cdot \\ \cdot \\ 1 \end{array}\right| \right\},$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = - \begin{bmatrix} 30+j7 & 16 & 26-j7 \\ 12-j4 & 38+j2 & 40-j8 \\ 4-j7 & 12-j4 & 38-j7 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} -14-j7 & -4-j14 & -10+j7 \\ 26+j6 & 28-j4 & -2+j10 \\ 8+j3 & 34 & -26+j3 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \cdot \\ \cdot \end{bmatrix}, \begin{bmatrix} \cdot \\ 1 \\ \cdot \end{bmatrix} \text{or} \begin{bmatrix} \cdot \\ \cdot \\ 1 \end{bmatrix} \Bigg\}$$

$$= \frac{1}{12400+j6156} \begin{bmatrix} 505+j3 & -162-j38 & -181+j43 \\ -162-j38 & 567+j133 & -486-j114 \\ -19+j81 & -162-j38 & 467+j195 \end{bmatrix} \left\{ \begin{bmatrix} -14-j7 \\ 26+j6 \\ 8+j3 \end{bmatrix}, \begin{bmatrix} -4-j14 \\ 28-j4 \\ 34 \end{bmatrix}, \begin{bmatrix} -10+j7 \\ -2+j10 \\ -26+j3 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 1 \\ -1 \\ \cdot \end{bmatrix}, \begin{bmatrix} 1 \\ \cdot \\ -1 \end{bmatrix}, \begin{bmatrix} \cdot \\ -1 \\ 1 \end{bmatrix}.$$

$$\therefore x_\kappa^b = \begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & \cdot \\ 2 & -1 & \cdot & -1 \\ 3 & \cdot & -1 & 1 \\ 4 & 1 & \cdot & \cdot \\ 5 & \cdot & 1 & \cdot \\ 6 & \cdot & \cdot & 1 \end{array}.$$

If we put $K_b^a = \begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{bmatrix}$,

$$B_\kappa^a = K_b^a x_\kappa^b = \begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & \cdot & 1 & \cdot & \cdot \\ 1 & \cdot & -1 & \cdot & 1 & \cdot \\ \cdot & -1 & 1 & \cdot & \cdot & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & \cdot & 1 & \cdot & \cdot \\ -1 & \cdot & 1 & \cdot & -1 & \cdot \\ \cdot & 1 & 1 & \cdot & \cdot & -1 \end{bmatrix}$$

$$D_\kappa^i = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & -1 & \cdot & 1 & \cdot & \cdot \\ 2 & -1 & \cdot & 1 & \cdot & -1 & \cdot \\ 3 & \cdot & 1 & 1 & \cdot & \cdot & -1 \\ \hline 4 & \cdot & \cdot & \cdot & -1 & 1 & 1 \end{array}.$$

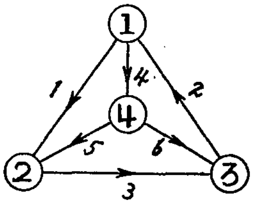

Fig. 4

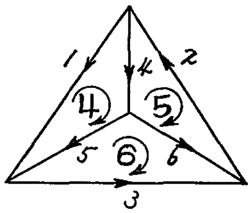

Fig. 5

Then we have the connection such as in Fig. 4. If we think independent loops such as in Fig. 5, the loop matrix of this network is given as

$$R_p^\kappa = \begin{array}{c|ccc|} & 4 & 5 & 6 \\ \hline 1 & -1 & \cdot & \cdot \\ 2 & \cdot & -1 & \cdot \\ 3 & \cdot & \cdot & -1 \\ 4 & 1 & -1 & \cdot \\ 5 & 1 & \cdot & -1 \\ 6 & \cdot & -1 & 1 \end{array}.$$

The generalized inverse of R_q^λ is given as

$$R_\kappa^p = \begin{array}{c|cccccc|} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 4 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 5 & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\ 6 & \cdot & \cdot & -1 & \cdot & \cdot & \cdot \end{array}.$$

Then on the one hand we have

$$y^{pq} = R^p_\kappa y^{\kappa\lambda} R^q_\lambda = \frac{1}{162+j38}\begin{vmatrix} 30+j7 & 16 & 26-j7 \\ 12-j4 & 38+j2 & 40-j8 \\ 4-j7 & 12-j4 & 38-j7 \end{vmatrix}.$$

$$\therefore\quad z_{qp} = (y^{pq})^{-1} = \begin{vmatrix} 6-j1 & -2 & -2+j \\ -2 & 7 & -6 \\ j1 & -2 & 6+j \end{vmatrix} = \begin{vmatrix} 6-j1 & -2 & -1+j \\ -2 & 7 & -4 \\ -1+j1 & -4 & 6+j \end{vmatrix} + \begin{vmatrix} \cdot & \cdot & -1 \\ \cdot & \cdot & -2 \\ 1 & 2 & \cdot \end{vmatrix}.$$

$$\therefore\quad z_{(qp)} = \begin{vmatrix} 6-j & -2 & -1+j \\ -2 & 7 & -4 \\ -1+j1 & -4 & 6+j \end{vmatrix}. \qquad (+A)$$

On the other hand

$$z_{(qp)} = R^\lambda_q z_{(\lambda\kappa)} R^\kappa_p = \begin{vmatrix} z_1+z_4+z_5 & -z_4 & -z_5 \\ -z_4 & z_2+z_4+z_6 & -z_6 \\ -z_5 & -z_6 & z_3+z_5+z_6 \end{vmatrix}. \qquad (+B)$$

Therefore comparing (+A) with (+B), we determine elementimpedancs. Namely,

$$z_1 = 3,\ z_2 = 1,\ z_3 = 1+j2,\ z_4 = 2,\ z_5 = 1-j1,\ z_6 = 4.$$

And also the unsymmetrical part of z_{qp} is

$$z_{[qp]} = \begin{vmatrix} \cdot & \cdot & -1 \\ \cdot & \cdot & -2 \\ 1 & 2 & \cdot \end{vmatrix}. \qquad (-A)$$

This valus is given, if gyrators are added in the above network such as in Fig. 6 (a) or (b). Namely

(a) $z_{51} = 1$, $z_{15} = -1$,
$z_{36} = 2$, $z_{63} = -2$,

(b) $z_{51} = 1$, $z_{15} = -1$,
$z_{26} = 2$, $z_{62} = -2$.

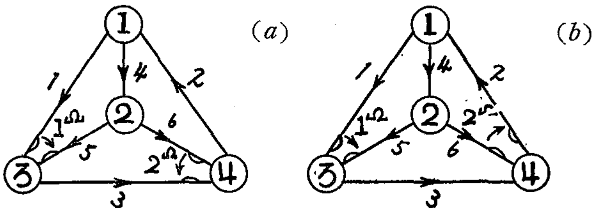

Fig. 6

Therefore if we adopt (a) in Fig. 6, we have the network such as in Fig. 7.

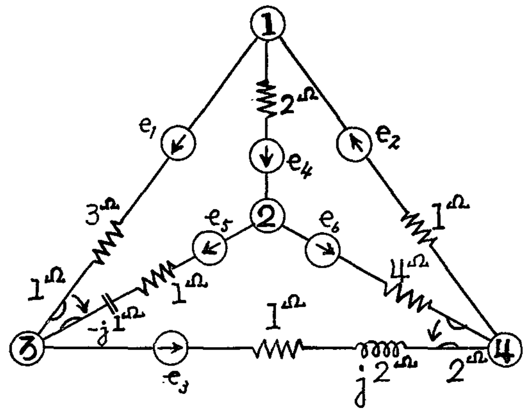

Fig. 7

4.2 **Problem 2**

Design a network specified by the driving admittance matrix $C^{\kappa\lambda}$ such as

$$C^{\kappa\lambda}=\frac{1}{167+j13}\begin{bmatrix} 52 & 40-j6 & 12+j6 & -14+j6 & -12-j6 & 38+j6 & -14+j6 & 24+j12 \\ 48+j2 & 50-j3 & -2+j5 & -26+j4 & 2-j5 & 22+j6 & -26+j4 & -4+j10 \\ 4+j2 & -10-j3 & 14+j1 & 12+j2 & -14-j1 & 16 & 12+j2 & 28+j2 \\ -22-j2 & -30 & 8-j2 & 19-j1 & -8+j2 & -3-j3 & 19-j1 & 16-j4 \\ -4+j2 & 10+j3 & -14-j1 & -12-j2 & 14+j1 & -16 & -12-j2 & -28-j2 \\ 30-j2 & 10-j6 & 20+j4 & 5+j5 & -20-j4 & 35+j3 & 5+j5 & 40+j8 \\ -22-j2 & -30 & 8-j2 & 19-j1 & -8+j2 & -3-j3 & 19-j1 & 16-j4 \\ 8-j4 & -20-j6 & 28+j2 & 24+j4 & -28-j2 & 32 & 24+j4 & 56+j4 \end{bmatrix},$$

SOLUTION First we determine a maximal set of independent solutions of $C^{\kappa\lambda}x_\lambda = O$ as follows:

$$\begin{bmatrix} 4-j2 & -10-j3 & 14+j1 & 12+j2 & -14-j1 & 16 & 12+j2 & 28+j2 \\ 30-j2 & 10-j6 & 20+j4 & 5+j5 & -20-j4 & 35+j3 & 5+j5 & 40+j8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = O.$$

$$\therefore \begin{bmatrix} 4-j2 & -10-j3 \\ 30-j2 & 10-j6 \end{bmatrix}\begin{bmatrix} x^1 \\ x_2 \end{bmatrix}$$

$$= -\begin{bmatrix} 14+j1 & 12+j2 & -14-j1 & 16 & 12+j2 & 28+j2 \\ 20+j4 & 5+j5 & -20-j4 & 35+j3 & 5+j5 & 40+j8 \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}.$$

Putting

$$\begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}, \begin{bmatrix} \cdot \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}, \begin{bmatrix} \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}, \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \end{bmatrix}, \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \cdot \end{bmatrix}, \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -\frac{3}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}.$$

Then we have

$$
x^{b}_{\kappa} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\left|\begin{array}{cccccc}
-1 & -\frac{1}{2} & 1 & -\frac{3}{2} & -\frac{1}{2} & -2 \\
1 & 1 & -1 & 1 & 1 & 2 \\
1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{array}\right|
\end{array} .
$$

If we put

$$
K^{a}_{b} = \left|\begin{array}{cccccc}
1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & -1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & 1 & -1 \\
-1 & \cdot & 1 & \cdot & \cdot & 1 \\
1 & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & -1 & \cdot
\end{array}\right| ,
$$

$$
B^{a}_{\kappa} = K^{a}_{b} x^{b}_{\kappa}
$$

$$
= \left|\begin{array}{cccccc}
1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & -1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & 1 & -1 \\
-1 & \cdot & 1 & \cdot & \cdot & 1 \\
1 & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & 1 & \cdot & \cdot & -1 & \cdot
\end{array}\right|
\left|\begin{array}{cccccccc}
-1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
-\frac{1}{2} & 1 & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\
1 & -1 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\
-\frac{3}{2} & 1 & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\
-\frac{1}{2} & 1 & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\
-2 & 2 & \cdot & \cdot & \cdot & \cdot & \cdot & 1
\end{array}\right|
$$

$$
= \left|\begin{array}{cccccccc}
-1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & 1 & \cdot & -1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & -1 \\
\cdot & \cdot & -1 & \cdot & 1 & \cdot & \cdot & 1 \\
\cdot & \cdot & 1 & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & -1 & \cdot
\end{array}\right| .
$$

Therefore we have

$$
D^{i_1}_{\kappa} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\left|\begin{array}{cccccccc}
-1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & \cdot & \cdot & 1 & \cdot & -1 & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & -1 \\
\cdot & \cdot & -1 & \cdot & 1 & \cdot & \cdot & 1 \\
\hline
\cdot & -1 & \cdot & -1 & -1 & \cdot & -1 & \cdot
\end{array}\right|
\end{array}
$$

and

$$
N^{a_2}_{\kappa} = \begin{array}{c} 5 \\ 6 \end{array}
\left|\begin{array}{cccccccc}
\cdot & \cdot & 1 & \cdot & 1 & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & -1 & \cdot
\end{array}\right| .
$$

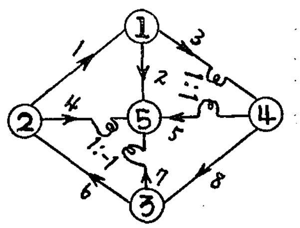

Fig. 8

These matrix $D^{i_1}_{\kappa}$ and $N^{a_2}_{\kappa}$ shows the network in Fig. 8.

A Maximal set of independent solutions for $B^{a_2}_{\kappa} x^{\kappa} = O$ are determined as follows:

$$
C^{\kappa}_{p} = \begin{array}{c|cc|} & 7 & 8 \\ \hline 1 & -2 & 2 \\ 2 & -3 & 2 \\ 3 & 1 & \cdot \\ 4 & 2 & -1 \\ 5 & -1 & \cdot \\ 6 & \cdot & 1 \\ 7 & 2 & -1 \\ 8 & 2 & \cdot \end{array} .
$$

Then the generalized inverse of C^{κ}_{a} is

$$
C^{\kappa}_{p} = \begin{array}{c|cccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 7 & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ 8 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \end{array} .
$$

On one hand,

$$
z_{qp} = (C^{p}_{\kappa} C^{\kappa\lambda} C^{q}_{\lambda})^{-1} = \left\{ \frac{1}{167+j13} \begin{bmatrix} 14+j1 & 16 \\ 20+j4 & 35+j3 \end{bmatrix} \right\}^{-1} = \begin{bmatrix} 35+j3 & -16 \\ -20-j4 & 14+j1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 35+j3 & -18-j2 \\ -18-j2 & 14+j1 \end{bmatrix} + \begin{bmatrix} \cdot & 2+j2 \\ -2-j2 & \cdot \end{bmatrix} .
$$

i. e.
$$
z_{(qp)} = \begin{bmatrix} 35+j3 & -18-j2 \\ -18-j2 & 14+j1 \end{bmatrix} , \qquad (+A)
$$

and
$$
z_{[qp]} = \begin{bmatrix} \cdot & 2+j2 \\ -2-j2 & \cdot \end{bmatrix} . \qquad (-A)
$$

On the other hand,

$$
z_{(qp)} = C^{\lambda}_{q} z_{(\lambda\kappa)} C^{\kappa}_{p} = \begin{bmatrix} -2 & -3 & 1 & 2 & -1 & \cdot & 2 & 2 \\ 2 & 2 & \cdot & -1 & \cdot & 1 & -1 & \cdot \end{bmatrix} \begin{bmatrix} z_1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & z_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & z_3 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & z_4 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & z_5 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & z_6 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & z_7 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & z_8 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ -3 & 2 \\ 1 & \cdot \\ 2 & -1 \\ -1 & \cdot \\ \cdot & 1 \\ 2 & -1 \\ 2 & \cdot \end{bmatrix}
$$

$$
= \begin{bmatrix} 4z_1+9z_2+z_3+4z_4+z_5+4z_7+4z_8 & -4z_1-6z_2-2z_4-2z_7 \\ -4z_1-6z_2-2z_4-2z_7 & 4z_1+4z_2+z_4+z_6+z_7 \end{bmatrix} . \qquad (+B)
$$

Comparing $(+A)$ with $(+B)$,

$z_1 = 1$, $z_2 = 2$, $z_3 = 1$, $z_4 = j1$,

$z_5 = -j1$, $z_6 = 1$, $z_7 = 1$, $z_8 = 2$.

By $(-A)$ we add the following gyrator in the above network:

$$
\begin{cases} z_{36} = 2+j2, \\ z_{63} = -2-j2. \end{cases}
$$

Finally the determined network is shown in Fig. 9 .

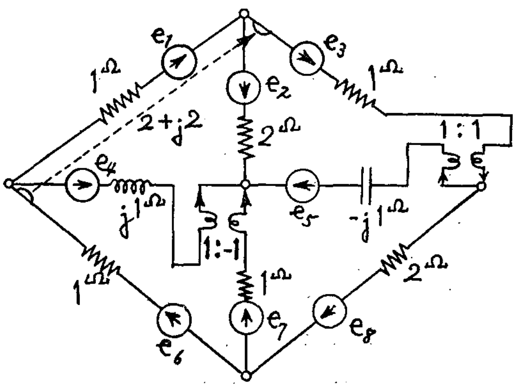

Fig. 9

5. APPENDIX

With regard to synthesis by using only self-impedances, they are determined uniquely for a connection if

$$n = \frac{k(k+1)}{2},$$

not if

$$n > \frac{k(k+1)}{2},$$

where n is the dimension of the given matrix, namely the number of self-impedances and k is the rank of the given matrix.

For example, the following sets of impedances respectively satisfy PROBLEM 2.

(a) $z_1=0$, $z_2=0$, $z_3=0$, $z_4=8$, $z_5=-1-j1$, $z_6=5$, $z_7=1+j1$, $z_8=0$.

(b) $z_1=0$, $z_2=0$, $z_3=-1-j1$, $z_4=8$, $z_5=0$, $z_6=5$, $z_7=1+j1$, $z_8=0$.

(c) $z_1=0$, $z_2=0$, $z_3=-\frac{1}{2}-j\frac{1}{2}$, $z_4=8$, $z_5=-\frac{1}{2}-j\frac{1}{2}$, $z_6=5$, $z_7=1+j1$, $z_8=0$.

With regard to gyrators, the values thereof is not unique generally and the example of them is shown in Fig. 6 of PROBLEM 1.

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ONODERA : Topological Synthesis of Non-Reciprcoal Networks

非可逆回路の位相幾何的構成

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邦文梗概 可逆回路の位相幾何学的構成については本学紀要(工学)第五巻第一号に発表したが，ここでは線形非可逆回路の構成法への拡張を試みた。この回路では伝達アドミツタンス行列は一般に非対称となるので，任意に与えられた非対称行列を伝達アドミツタンスとするような回路を実現する方法について述べた。対称成分は抵抗，インダクタンス，キャパシタンスで交代成分はジャイレータで与えるのは他の方法と同様であるが，接続及び理想変成器の決定には与えられた行列の左右両側に同じ実数零因子を有することが条件で，その実数零因子の非特異変換(線形)によって得た行列の一部が接続行列となり即ち接続が決定され，他の一部が変成器の巻数行列となり即ち理想変成器の使用が決定される。トランジスタ回路，真空管回路等は一般の素子とジャイレータで表現できるのでそれらの構成法にもある特別な場合には応用できる。但し，普通の素子とジャイレータ素子とからできる回路はトランジスタ又は真空管回路にそのまゝ変えるにはその性質上接続に特殊の条件が入るのでその間の変換は又別の問題である。